in ballistic ranges. Profiles of the radiation intensity across the wake may be made every few body diameters with an optical resolution of a small fraction of a body diameter. The data from the instrument have been used to reconstruct radiation histories, measure intensity decay, and measure the growth of luminous wakes.[8] Because of the greater sensitivity of photoelectric devices relative to photographic film, the wake scanner can provide luminous growth data farther downstream in the wake than the race track technique.[9] By using optical filters, the wake scanner can provide spectral as well as time and spatial resolution, yielding further information on the chemistry and temperature of the wake.

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Compressible Jet Spread Parameter for Mixing Zone Analyses

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By a representation of reference density and mixing zone density ratio, a new formulation of the compressible jet spread parameter is attained. This analytic approximation is shown to be in good correlation with the existing experimental data. The use of the present formulation in the mixing zone analysis is believed to yield a better fit of the analytical results with experimental data in the high Mach number range.

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Nomenclature

c = Tollmien constant
C = Crocco number
D = divergence constant of an incompressible jet
l = mixing length
M = Mach number
R = compressible divergence factor
T = temperature
ρ = density
τ = shear stress
$\bar{\alpha}$ = constant for a given Crocco number
β = stagnation temperature ratio, T_{01}/T_{02}
σ = jet spread parameter
γ = specific heat ratio

Subscripts

r = some reference condition
0 = stagnation condition
$*$ = transformed or incompressible value
1 = freestream or jet
2 = secondary stream or still air region

THE jet spread factor σ, often used in the analysis of jet mixing and base heating problems, has been formulated in the past on engineering intuition and experimental verification. However, so far no rigorous analysis or basis has been presented [except for the experimental values (*) in the thus arrived at value of σ]. Korst and Tripp[1] have taken the first step in formulating an empirical relation for the compressible jet spread parameter ($\sigma = 12 + 2.76M$).

In the absence of any relevant theory, this note presents a semiempirical relation that is believed to be an improvement on the existing values and formulations of the compressible jet spread parameter. From a comparison with the experimental data, it is observed that the present approach does show the correct trend.

Phenomenological Model for a Compressible Jet Spread Parameter

Various phenomenological models for turbulent fluxes have been presented in the past, and almost all of them hinge upon the unknown variable density of the dissipative region. The density ratio (ρ_r/ρ) has been represented in terms of the mean properties of the flow, with some knowledge of predicting these distributions in the mixing zone. Such formulations can be checked only against experimental data.

Because of lack of experimental data for flows at high Mach numbers, authors have resorted to extrapolation of values of jet spread parameter.[3] Some results of the empirical formulations and extrapolated values of σ are shown to have been in good correlation with experimental data.[1, 3]

In view of the foregoing, this note presents a formulation of the compressible jet spread parameter based on a representation of the density ratio with the mean flow properties. Here, free jet mixing into still air will be considered.

From Prandtl's mixing length hypothesis, the shear stress in a turbulent flow is given by

$$\tau = \rho l^2(\partial u/\partial y)^2 \tag{1}$$

Using Howarth[2] transformation, one has for shear stress τ_*, in the transformed plane,

$$\tau_* = \rho_r l_*{}^2(\partial u/\partial y_*)^2 \tag{2}$$

The transformation is essentially stretching of the y coordinate and is defined by

$$\begin{aligned} \frac{\partial}{\partial x} &= \frac{\partial}{\partial x_*} + \frac{\partial y_*}{\partial x}\cdot\frac{\partial}{\partial y_*} \\ \frac{\partial}{\partial y} &= \left(\frac{\rho}{\rho_r}\right)\frac{\partial}{\partial y_*} \end{aligned} \tag{3}$$

Following Mager's[4] assumption (the invariancy of shear

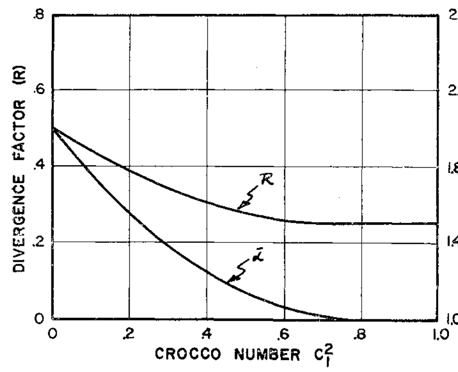

Fig. 1 Variation of compressible divergence factor with Crocco number.

stress under transformation), upon transforming Eq. (1) and comparing with Eq. (2), one has

$$l^2 = (\rho_r/\rho)^3 l_*^2 \tag{4}$$

The mixing length l is written as

$$l = x/[2\sigma^3]^{1/2} \qquad (c^2 = 1/2\sigma^3 \text{ Tollmien's notation})$$

Since $x = x_*$,

$$\sigma = \sigma_*/(\rho_r/\rho) \tag{5}$$

In order to compute the value of compressible jet spread parameter σ, the density ratio of Eq. (5) is to be represented in terms of known flow properties.

Now the density ratio can be represented as

$$(\rho_r/\rho) \backsim [(\rho_1 + \rho_2)/\rho_1]$$

or

$$(\rho_r/\rho) = R[(\rho_1 + \rho_2)/\rho_1] \tag{6}$$

A compressible divergence factor R (see Fig. 1) is a function of the Crocco number and defined as

$$R = D\bar{\alpha}$$

where $D = 0.25$ is a divergence constant for an incompressible jet,[5, 6] and $\bar{\alpha}$ is a computed mean (Fig. 1) dependent on Crocco number.[8, 9]

Hence, one can now express the jet spread parameter ratio as

$$\frac{\sigma}{\sigma_*} = \left[R\left(\frac{\rho_1 + \rho_2}{\rho_1}\right)\right]^{-1} \tag{7}$$

For constant pressure condition, one can express Eq. (7)

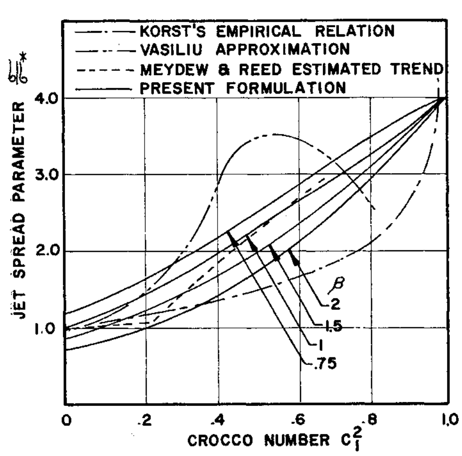

Fig. 2 Variation of jet spread parameter with Crocco number.

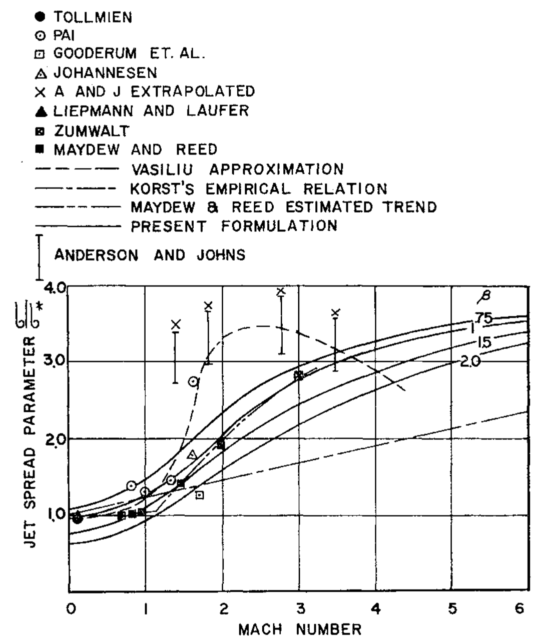

Fig. 3 Variation of jet spread parameter with Mach number.

in terms of the Mach number of the jet and stagnation temperature ratio:

$$\sigma/\sigma_* = \{R[1 + (\beta/Z)]\}^{-1} \tag{8}$$

where

$$\beta = T_{01}/T_{02}$$

$$Z = 1 + [(\gamma - 1)/2]M_1^2$$

or, in terms of the Crocco number,

$$\sigma/\sigma_* = [R\{1 + \beta(1 - C^2)\}]^{-1} \tag{9}$$

Plots of Eqs. (8) and (9) (Figs. 2 and 3) show good correlation with the experimental data in the low Mach number range as presented by Vasiliu[3] and the data obtained by Maydew and Reed.[7] The present formulation is particularly in good agreement with Maydew and Reed's estimated trend and Zumwalt's data. The experimental data of Zumwalt include the correction for virtual origin.[10]

From Fig. 3 it is predicted that the spread parameter variation has an upper limit ($\sigma/\sigma_* = 4$) as the Crocco number approaches unity. The present model predicts a strong influence of the thermal level on the rate of spread of the mixing zone. Also, for a given Mach number or Crocco number, the hotter the jet, the lower the value of σ/σ_* and, hence, the greater the spread of the mixing zone. For the two parallel stream mixing cases, a jet spread parameter can be formulated in a similar manner.

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Continuum Aspects of rf Gradient Acceleration of Plasma

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A quasi-one-dimensional model of acceleration of a continuum plasma by rf electric field gradient forces is examined. The conditions imposd on the electric field in order to effect sonic transitions are given. The case of isentropic flow is treated, and departure from the particle model is indicated for pressure >100 μ and temperatures ≃0.1 ev.

Nomenclature

A	=	cross-sectional area of channel
c	=	sonic speed
c_p	=	specific heat at constant pressure
c_v	=	specific heat at constant volume
D	=	electric displacement flux
E	=	electric field intensity
E_a	=	applied electric field
F_{em}	=	force/unit volume
K	=	dielectric constant
L	=	system constant
l	=	e^2/mM'
M	=	sonic Mach number = v/c
M'	=	ion mass
m	=	electron mass
p	=	kinetic pressure
Q	=	specific heat input
R	=	universal gas constant
T	=	temperature
t	=	time
v	=	velocity
x	=	axial coordinate
β	=	constant in adiabatic gas law
γ	=	specific heat ratio
ϵ	=	permittivity of free space
λ	=	fractional ionization
μ_0	=	$\rho_0 v_0$ = initial momentum
ρ	=	mass density
σ	=	electrical conductivity
τ	=	$\beta\mu_0{}^{\gamma-1}[\gamma/(\gamma-1)]$
ω	=	applied frequency
ω_p	=	plasma frequency = $(\lambda\rho e^2/\epsilon m M')^{1/2} = (l\rho)^{1/2}$

Acceleration of an electrically neutral plasma by application of a spatially inhomogeneous nonpropagating rf electric field has been demonstrated experimentally[1] at 140, 330, and 2424 Mc. Theoretical studies,[2] using a particle model of the plasma, have predicted the final or exit velocity in terms of the field applied.

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The expression for the time-averaged acceleration on the basis of this model is

$$\ddot{x} = \frac{e^2}{4mM'}\frac{1}{\omega^2 - \omega_p{}^2}\nabla E_a{}^2 \tag{1}$$

However, in some of the forementioned experiments it would appear appropriate to apply continuum analysis for certain portions of the accelerating regions because of the size of the mean free path involved. In this paper a one-dimensional channel flow is examined in which the forces are time-averaged. The conditions for sonic transition are stated, and for the case of isentropic flow the exit velocity is given in terms of the electric field maximum.

Analysis of the Flow

In the presence of electric and magnetic fields an electrically conductive fluid is subjected to a body force. The expression for such a force density can be obtained by variation of the electromagnetic free energy integral.[3]

Those terms in the expression for the force density which are relevent to the purposes at hand are as follows:

$$F_{em} = \frac{\epsilon}{2}\nabla\left(E^2\rho\frac{dK}{d\rho}\right) - \frac{\epsilon}{2}E^2\nabla K \tag{2}$$

The specific form of the terms in (2) is, in part, because the variation was performed isothermally. Hence, rigorously speaking, thermal energy changes in the fluid which accompany the field changes are not considered. Therefore, the variation from which the expression for the force density is obtained corresponds only to the variation of the available or electromagnetic "free" energy. Nonetheless, if changes in the value of the dielectric constant which are due to temperature are small, then their effects can be ignored in the expression F_{em}. At the same time, the effects of temperature changes may be included in the energy conservation statement.

The temporal variation in the problem to be considered is avoided by using time-averaged quantities. This treatment is justified by the fact that the ionic constituent of the plasma is relatively motionless during a period corresponding to one cycle. At the same time, however, the field does induce a net electronic displacement and hence gives rise to the polarization of the medium which reacts with the field.

The one-dimensional form of the conservation equations can be written as

$$\rho v A = \text{const} \tag{3}$$

$$\rho v\frac{dv}{dx} + \frac{dp}{dx} = \frac{\epsilon}{2}(K-1)\frac{dE^2}{dx} \tag{4}$$

$$\rho c_p v\frac{dT}{dx} + \rho v^2\frac{dv}{dx} + \frac{v}{2}\frac{d}{dx}(E\cdot D) = Q + \sigma E^2 \tag{5}$$

In Eq. (4), the explicit form for the expression F_{em} has been inserted. This is justified on the following basis. K is assumed to be that expression which is derivable on the basis of a linear harmonic analysis, that is, $K = 1 - \alpha\rho$, where $\alpha = l/\omega^2$. This is probably quite good except at frequencies very near the resonance of the plasma. Here it is expected that nonlinear effects become important. Even so, K† must change sign in going from one side of the resonance to the other, and this is the important fact relevent to the transition.

† Physically, K goes positive when the applied frequency goes higher than the plasma frequency, because the phase of the electronic motion falls sufficiently far behind. The result of this is that the polarization of the induced field becomes parallel to that of the applied field.